Multi-Dimensional Development – An Application of Fuzzy Set Theory to the Indian States

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14 February 2017

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Abstract

Even though India has recently become one of the fastest growing economies of the world and one among the most important G-20 economies, in terms of many development indicators, India has not fared well. Ours is a country of wide diversity in regional, social, economic, political, and cultural dimensions. Different States with different policy mixes have witnessed very different outcomes over the years. Some States have focused only on growth and some States have won laurels in achieving the objectives of both growth and development simultaneously. Analysis of this diversity and disparity across the States in their performance would help us identify useful policies of development.

However, many concepts/predicates, such as poverty (or poor) and its opposite, development (or developed), used in economics are both vague/fuzzy and multi-dimensional and their analysis requires careful consideration of a graded membership. This study therefore employs the framework of fuzzy set theory in identifying and analysing the positions of different states in the development ladder, that is, their graded memberships in each development dimension and in aggregation.

The development dimensions that we consider are health, knowledge and standard of living. Note that these dimensions are latent factors, that is, unobservable; hence we have to use some indicators to proxy these development dimensions. The indicators of health dimension are: (i) Life expectancy at birth, (ii) Infant mortality rate, (iii) Birth rate, and (iv) Death rate. As an indicator of knowledge we take literacy rate, and that of standard of living, per capita net state domestic product at constant prices prices. The selected states are: Andhra Pradesh, Assam, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal. The data have been sourced from the Planning Commission of India (http://planningcommission.nic.in/data/datatable/).

ACKNOWLEDGEMENT

The present work is an earlier version of the Project Report entitled "A Multi-Dimensional Assessment of Development – An Application of Fuzzy Set Theory to the Indian States", submitted as part of the requirements for the award of BA Degree in Economics of University of Madras, Chennai, in 2017.

This project would not have been completed without the encouragement and help of several people. I would like to thank each of them for their help, inspiration and cooperation.

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14 February 2017.

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Chapter 1
Introduction

> "Vagueness, … is my topic tonight ….. You will no doubt think that, in the words of the poet: "Who speaks of vagueness should himself be vague". I propose to prove that all language is vague, and that therefore my language is vague, but I do not wish this conclusion to be one that you could derive without the help of the symbolism. I shall be as little vague as I know how to be if I am to employ the English language."
>
> – Bertrand Russel (1923: 84)

1.1 Prologue

Let us start with the famous 'paradox of the heap' (more properly called *sorites paradox*; the word 'sorites' is a Greek word meaning heap; the paradox is attributed to Eubulides of Miletus of 4th century BC). The paradox involves a heap of sand, from which grains of sand are removed one by one. We know that when we remove a single grain, what remains is still a heap; remove one more, still a heap remains; one more, again one more, we will still have a heap; now what happens when we extend this logic as we remove more and more individual grains such that each time a grain of sand is removed we are still left with a heap of sand? When we are finally left with a single grain, the logic suggests that we still have a heap! Thus we are led to a contradiction. That is the paradox. But when can we say the heap ceased to be a heap and turned into a non-heap?

We have a number of similar paradoxes. Consider a bag of some sand grains that I can very easily lift. Add one more grain of sand, I can still lift that bag; one more, again one more; still I can lift it easily. If we continue with this logic, it can mean that I will be able to lift the bag of sand grains, even if it has tons of sand! Another paradox, called the bald man (*phalakros*) paradox runs like this: suppose we have a friend with a head full of hair; removing a single hair will not turn him into a bald man; remove one more hair, again one more; he will still be non-bald. If we extend this logic repeatedly, what will happen? We will have to admit that our friend is still non-bald, even if there is only one hair finally on his head! A contradiction.

Such paradoxes are collectively called the continuum fallacy. It refers to vagueness or uncertainty surrounding many concepts and predicates and argues that two states or conditions cannot be considered distinct, because between them there exist different states in a continuum. For example, consider the predicates such as 'tall' and 'nice', adverbs such as 'quickly' and even quantifiers such as 'many', all of which can be vague (Keefe and Smith 1996, p 5). So are the predicates 'developed' and 'poor', or the concepts of 'development' and 'poverty'. Before coming to these vague predicates/concepts, let us consider the vague predicate 'tall'.

We know that certainly there are cases in which we can without doubt classify people as 'tall' and 'short', even though we do not have any exact borderline of height *h*, above which one is tall and below which one is short. At the same time, there are certain cases in which we cannot state that a person is *definitely* tall. In such cases, we may say that person is 'borderline tall'. Also note that the sorites paradox too applies here. Suppose Ram is definitely tall. Now consider cutting away a millimeter of his height; he will still be tall. Cut away one more millimeter, again another millimeter, he will still be tall. Extending this logic leads us to the usual contradiction that he will still be tall, even after a sizeable height is cut away from him.

Similarly, the predicate 'poor' also is vague; so are the related predicates 'extreme' and 'chronic', used in the measurement of poverty. For example, take the case of a person, poor in income. We know that giving her one more paise (one-hundredth denomination of a rupee) will not make her non-poor. Give her one more paise, again one more paise, still she will not be non-poor. Extending this logic several times will lead us to the usual contradiction that she will still be poor, even after amassing a lot of income. Similarly, we might come across several cases in which we might classify a person as 'borderline poor'. Moreover, the official use of poverty line to distinguish between the poor and the non-poor cannot be taken as exact, because it is not possible to find a sharp borderline between the poor and the non-poor. For example, take the poverty line accepted in India in terms of a benchmark daily per capita expenditure of Rs. 27 and Rs. 33 in rural and urban areas, respectively. Can we say a rural person having a daily per capita expenditure of Rs. 27.01 is non-poor? Can we so assert, even if we increase her income to Rs. 27.50 or even to Rs. 28 or 29? Thus the predicate 'poor' is as vague as 'tall'. The same applies to the other predicates relevant to poverty measurement such as 'extreme' and 'chronic'. Note that these are only a few among many vague predicates/concepts that social scientists usually study.

Thus a concept or predicate may be considered *vague* (i) if it is found to be lacking in clarity, or (ii) if there is uncertainty about the kind of objects belonging to that concept or having characteristics that correspond to that predicate (so-called 'border-line cases'), or (iii) if the Sorites paradox applies to the concept or predicate.

In explaining vagueness, so far we have considered examples having only one dimension. In the case of the predicate 'tall', we have taken height as the only dimension. We have explained baldness in terms of the number of hairs on the head of a person. And in the case of poverty, the only dimension we have considered is the amount of income one has. However, in the case of some concepts or predicates, we can find that multiple dimensions are relevant. For example, consider the predicate 'nice'. We know that our friend Rani is very polite, sociable and generous, but is sometimes bad-tempered also. We may say that Rani is not *definitely* nice, but only borderline nice, considering all the relevant (multiple) dimensions.

So is the predicate 'poor', which is multi-dimensional as well as vague. A person may be poor not just in terms of income; she may be poor in health, education, etc. also. Now suppose that Rani has a good income, but is illiterate and chronically ill. She is classified as non-poor in terms of income, but as poor in terms of education and health, even if we allow for vagueness of 'poor' in these dimensions. Now can we consider her 'poor'? Or suppose that Rani has a very low income, but is well educated and very healthy; thus she is poor in terms of income, but non-poor in health and education (after allowing for vagueness of 'poor' in these dimensions). Now can we consider her 'poor'?

From a multi-dimensional viewpoint of poverty, it is not clear whether we can classify her to be poor or non-poor in these cases. So she may be classified as 'borderline poor' in both these cases. Thus the multi-dimensionality of poverty is thus relevant to its vagueness.

Approaches to Poverty Measurement: Fuzzy Set Theory

So far we have considered predicates associated with individuals; the same applies to society or country. When we say a country is poor, we acknowledge that both the multi-dimensionality and vagueness of poverty are relevant here too. Its significance is evident from the fact that substantial "space and attention have also been dedicated to multidimensional poverty measurement in both developing and developed countries, where it has not only been studied theoretically, but also applied empirically." (Lemmi, and Betti, 2006: 1-2). In this context, there have been two most important types of approach:

i) one represented by theoretical models; and
ii) a second which builds on multivariate statistical methodology (such as discriminant analysis, factor analysis, cluster analysis, and correspondence analysis) and attempts to aggregate the basic information in terms of indicator vectors (*ibid.*).

An approach that belongs to category (i) comes from the mathematical theory of the fuzzy set logic, proposed by Zadeh (1965) and developed by Dubois and Prade (1980). The classical Logic has only two truth values: 'true' and 'false', so that the 'principle of bivalence' (which only allows for 'true' or 'false' statements) holds; these truth values may be represented by '1' for 'true' and '0' for 'false'. Thus it cannot consider cases of vagueness. A well-known account of vagueness comes in terms of 'degree theory' that drops classical Logic. The degree theory proposes a gradual transition between 'perfect falsity' to 'perfect truth', so that there are more than two truth values, that is, an infinite number of truth values along a spectrum between perfect truth and perfect falsity. Thus truth comes in degrees. Fuzzy set logic, developed by the American mathematician Lotfi Aliasker Zadeh (1965, 1975), seeks to quantify the degree of truth in borderline cases. Thus perfect truth may be represented by '1' and perfect falsity by '0', with borderline cases having a truth value anywhere between 0 and 1.

A workable empirical method of this approach, called Totally Fuzzy and Relative (TFR) method, was developed by Cheli and Lemmi (1995), starting from an idea of Cerioli and Zani (1990). "Further contributions (either for implementing the robustness and for considering the dynamic of poverty) have strongly improved the initial proposal and many applications have been undertaken in several economic and political realities at international, national and regional level, for developed, developing and less developed countries." (Lemmi, and Betti, 2006: 2). Moreover, a large number of studies (for example, Clark and Qizilbash 2002; Chiappero-Martinetti 2000; Lelli 2001) have shown that the fuzzy set approach is strictly consistent with the Capability theory of Amartya Sen.

Development, Capability Approach and Fuzzy Set Theory

It goes without saying that the term development is the opposite of poverty, and hence that concept also is vague and multi-dimensional. This term evades a unique definition and means different things to different economists. Todaro and Smith (2015: 18) emphasizes that development as a multidimensional process involves "major changes in social structures, popular attitudes, and national institutions, as well as the acceleration of economic growth, the reduction of inequality, and the eradication of poverty." According to them, "No one has identified the human goals of economic development as well as Amartya Sen, perhaps the leading thinker on the meaning of development." (*ibid*.).

Sen argues that it is not possible to properly measure poverty in terms of income or even by utility as usually understood; what matters fundamentally is not the things a person has—or the feelings these provide—but what a person *is,* or *can be*, and what she *does*, or *can do.* These beings and doings are called 'functionings', that is, what a person does (or can do and/or can be) with the commodities of given characteristics that she comes to possess or control. The valued beings and doings, or functionings that people have reason to value, can range from being healthy, being educated, being well-nourished, and well-clothed, to being mobile, having self-esteem, and “taking part in the life of the community.” (Sen, 1985: 12). He then defines ‘capability’ as “the freedom that a person has in terms of the choice of functionings, given his personal features (conversion of characteristics into functionings) and his command over commodities.” (Sen, 1985: 13). Thus alternative combinations of such functionings from which the individual can choose, in turn, define her capability. “Capability is thus a kind of freedom: the substantive freedom to achieve alternative functioning combinations (or, less formally put, the freedom to achieve various lifestyles).” (Sen 1999: 75)., or, “the range of options a person has in deciding what kind of life to lead.” (Dreze and Sen 1995: 10-11).

An individual’s freedom to promote the aspirations she has reason to value depends on her capability to achieve functionings that make up her wellbeing. In this sense, her freedom enhances with her capability set. Development is the process of enhancing freedom, expanding capability set, opportunities and choices “so that each person can lead a life of respect and value.” (UNDP 2000: 2). In other words, “Development consists of the removal of various types of unfreedoms that leave people with little choice and little opportunity of exercising their reasoned agency. The removal of substantial unfreedoms, …, is *constitutive* of development” (Sen 1999: xii). These freedoms are both the primary ends and principal means of development (Sen 1999: 10).

Sen’s capability approach helps explain why development economists have placed so much emphasis on health and education, and more recently on social inclusion and empowerment, and why they have referred to countries with high levels of income but with poor conditions of health and education as cases of ‘growth without development’ (See, for example, Easterly 2003).

Thus development is multi-dimensional and as vague as poverty and thus fuzzy. The present study is to analyse, in the fuzzy set theory framework, the multi-dimensional characteristics of development in some selected States in our country.

1.2 Need and Scope of the Study

Our country India has now achieved the distinction of being one of the fastest growing economies of the world and one among the most important G-20 economies. However, in terms of many development indicators, India has not fared well. As pointed out by Jean Dreze and Amartya Sen (1995: 78) "Four decades of allegedly interventionist planning did little to make the country literate, provide wide based health services, achieve comprehensive land reform or end the rampant social inequality that blight the material prospects of the underprivileged". Indian planners and policymakers have already felt this concern, as is evident in the 11th Five Year Plan document of 'Inclusive Growth'.

India is a country of wide diversity in regional, social, economic, political, and cultural dimensions. Different regions with different strategies and policy mixes have witnessed very different outcomes over the years. Some states have focused only on growth and some states have won laurels in achieving the objectives of both growth and development simultaneously. Analysis of this diversity and disparity across the states in their performance would help us identify useful policies of development. Hence the present study makes a novel attempt to analyse the issues of important aspects of development on a comparative plane across the states.

However, as we have already seen, many concepts/predicates, such as poverty (or poor) and its opposite, development (or developed), used in economics are both vague/fuzzy and multi-dimensional and their analysis requires careful consideration of a graded membership. This study therefore employs the framework of fuzzy set theory in identifying and analysing the positions of different states in the development ladder, that is, their graded memberships in each development dimension and in aggregation.

1.3 Objective of the Study

The main objective of the study is to analyse the important dimensions of development on a comparative plane across the Indian states in the framework of fuzzy set theory.

In particular, the objectives of the study are

(i) to review the methodologies of classical set theory and fuzzy set theory;

(ii) to identify the possible and empirically available dimensions of development of the Indian States; and

(iii) to analyse these important dimensions of development on a comparative plane across the Indian states in the framework of fuzzy set theory.

1.4 Data and Methods of Analysis

Development is a multi-dimensional concept, but imprecise and incomplete in terms of the dimensions that reflect particular states of development. Even though theoretically it is possible to expand the list of identified possible dimensions, empirically data on these aspects might not be completely available for all the states. Hence only a few major states and a few main dimensions of development are considered, where secondary data are readily available. The development dimensions considered are health, knowledge and standard of living. Note that these dimensions are latent factors, that is, unobservable; hence some indicators to proxy these development dimensions are to be used. The indicators of health dimension are as follows (along with the latest year for which data are available):

(i) Life expectancy at birth (2006-10),

(ii) Infant mortality rate (2012),

(iii) Birth rate (2012), and

(iv) Death rate (2012).

Literacy rate is taken as an indicator of knowledge, and per capita net state domestic product at constant prices as that of standard of living:

(v) Literacy Rate (2011), and

(vi) Per capita net state domestic product at constant (2004-05) prices (as on 31.10.2014); the last one represents average for three years from 2010-11. In order to iron out the possible fluctuations, a three-year average for per capita net state domestic product is used.

The selected states are: (i) Andhra Pradesh, (ii) Assam, (iii) Bihar, (iv) Gujarat, (v) Haryana, (vi) Karnataka, (vii) Kerala, (viii) Madhya Pradesh, (ix) Maharashtra, (x) Orissa, (xi) Punjab, (xii) Rajasthan, (xiii) Tamil Nadu, (xiv) Uttar Pradesh, and (xv) West Bengal.

Also note that Bihar, Madhya Pradesh and Uttar Pradesh are taken as undivided states, including the new states of Jharkhand (with Bihar), Chattisgarh (with Madhya Pradesh) and Uttarakhand (with Uttar Pradesh).

The data have been sourced from the Planning Commission of India (http://planningcommission.nic.in/data/datatable/).

The data are analysed in the framework of fuzzy set theory. A fuzzy set has as its base a classical set itself, but what distinguishes the two is the membership function, which *ranges* from 0 to 1 in the case of the fuzzy set, whereas the membership function of the classical set is constrained to either 1 (full membership) or 0 (non-membership). In the former (fuzzy set) case, degree of membership increases in proportion to its proximity to 1. Thus, the main difference between classical set theory and fuzzy set theory is that the latter allows for graded set membership. The present study is an attempt to estimate the partial membership function of different States in India in respect of a number of development indicators.

For want of acceptable benchmark values for all the indicators, we have used the maximum/minimum values in the data as cut-offs to identify the cases of definite membership and definite non-membership; the intermediate membership function values are derived using the simple linear model.

1.5 Limitations of the Study

The main limitation of the study comes from the non-availability of data on all the possible dimensions of development for all the States in India. Depending upon the available data, we have considered only six dimensions of development and 15 major States in India.

1.6 Organization of the Study

The present study is organized in five chapters. Following this chapter that has provided an introduction to the study problem, its objective, and data and methods of analysis, the next chapter presents a review of literature in respect of application of fuzzy set theory in studies of multi-dimensional poverty/development. Chapter 3 gives an introduction to fuzzy set theory, distinguishing it from the classical set theory and discussing various methods of estimating membership function. The empirical exercise follows in Chapter 4 that attempts to analyse, in the framework of fuzzy set theory, the comparative performance of different Indian States in achieving development. The final chapter summarises and concludes the study.

Chapter 2:

A Review of Literature

Driver: “Do I turn left?”

Passenger: “Right”.

2.1 Prologue

Fuzziness can be found anywhere in the real world. Adjectives such as beautiful, tall, young, poor, and concepts like poverty, development, satisfaction are all fuzzy. A major concern in modeling the real world around us is accommodating and treating such vague and imprecise information. Fuzzy set theory is about modeling such vagueness/uncertainty. Probability theory in statistics has been for a long time the major theory and tool to model uncertainties of reality. Fuzzy set theory is one of a number of theories that have come up as an alternative. Both fuzzy set theory and probability theory describe uncertainty in a numerical manner, using numbers in the unit interval [0, 1]. However, there are differences; fuzziness occurs when, and only when, the first law of Aristotle’s ‘Laws of Thought’ of non-contradiction is violated: “Classical logic and set theory assume that the law of non-contradiction and equivalently the law of excluded middle is never violated. That is what makes the classical theory black and white. Fuzziness begins where Western logic ends.” (Kosko 1990: 212-213).

Most of the traditional tools that we use for formal reasoning, modeling, and estimation are deterministic, and precise or crisp in character, where ‘crisp’ means binary, that is, yes-or-no type. For example, in traditional logic, a statement can be true (yes) or false (no); nothing between the two extremes is allowed. Thus, in set theory, an element can either belong to a set or not; in optimization, a solution can be feasible or not.

Against this, Zadeh, who first proposed the fuzzy set theory, wrote: “The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.” (Zadeh, 1965: 339). What Zadeh meant here by ‘imprecision’ is the sense of vagueness rather than the lack of knowledge about the value of a parameter. Zimmermann in his 2010 review states that “Fuzzy set theory provides a strict mathematical framework (there is nothing fuzzy about fuzzy set theory!), in which vague conceptual phenomena can be precisely and rigorously studied. It can also be considered a modeling language, well suited for situations in which fuzzy relations, criteria, and phenomena exist. The acceptance of this theory grew slowly in the 1960s and 1970s of the last century. In the second half of the 1970s, however, the first successful practical applications in the control of technological processes via fuzzy rule-based systems, called fuzzy control (heating systems, cement factories, etc.), boosted the interest in this area considerably. Successful applications, particularly in Japan, in washing machines, video cameras, cranes, subway trains, and so on triggered further interest and research in the 1980s so that in 1984 already approximately 4000 publications existed and in 2000 more than 30,000.” (Zimmermann 2010: 318).

2.2 Application of Fuzzy Set Theory in General

Since its inception in 1965, the fuzzy sets have advanced in both mathematical theory and applications in many disciplines in many ways. Applications of this theory are found profusely in, for example, artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, and robotics. In 1992, the fuzzy set theory combined with the theory of neural nets and the area of evolutionary programming to give birth to a new area, known under the name of ‘computational intelligence’ or ‘soft computing’.

Fuzzy logic is a direct outgrowth of fuzzy set theory and has become the base for many applications in fuzzy inference and control systems, ranging from simple static database structures to complex dynamic systems. Such developments include systems used for regional planning (Bárdossy and Duckstein 1995); dynamic inference systems that model foreign policy decision making (Seitz, 1994), and organizational behavior (Seitz, Hulin, and Hanisch, 2001); fuzzy data reduction techniques, initially motivated by research on pattern recognition, especially in fuzzy clustering methods (see Smithson, 1987, Ch. 5 for a review of work in the 1970s and 1980s), and in fuzzy clusterwise regression technique (Steenkamp and Wedel 1991); “grade of membership” extension of latent class analysis, which permits partial membership in the latent classes (Manton, Woodbury, and Tolley 1994; a computer program implementing their techniques, DSIGoM, is commercially available (from Decision Systems Inc.), and has been used in health studies and demography); and multilevel models that develop fuzzy sets for modeling household data structures where households change composition over time (Goldstein, Rasbash, Browne,Woodhouse, and Poulain (2000).

2.3 Application of Fuzzy Set Theory in Social Sciences

It can be noted that most of the applications are still in the industrial world, mainly in Japan and Germany where fuzzy technology is on the rise with fuzzy tools and fuzzy products such as video cameras, pattern recognition devices etc. (Zimmermann 1993). However, in the case of social sciences, fuzzy logic is still in infancy. Social sciences are said to be inexact science, since they all deal mainly with human behavior that is just uncertain and vague. Smithson has once remarked that “the human sciences tend to be methodologically conservative when mathematically sophisticated, and mathematically ignorant when methodologically innovative.” (Smithson, 1988: 2). He continues that “qualitatively oriented researchers are fond of castigating quantitative researchers for their inability to convincingly translate sophisticated theories of human behavior into mathematical form...while quantitative proponents berate ‘anti-positivists’ for the vagueness of their concepts and techniques." (Smithson, 1988: 12). It then appears that fuzzy set theory has great value in that it can bring the two warring camps together.

The term of 'fuzzy economics' was first used in the summer of 1985 at the First International Fuzzy System Association Congress held at Palma of Mallorca (Ponsard and Fustier 1986). Psychology has now fuzzy set-based theories of perception (Oden and Massaro, 1978) and memory (Massaro, Weldon, and Kitzis, 1991); and fuzzy-set based solutions for measurement problems and novel data analysis tools (Hesketh, Pryor, Gleitzman, and Hesketh, 1988; Parasuraman, Masalonis, and Hancock, 2000; Smithson, 1987; Wallsten, Budescu, Rappoport, Zwick, and Forsyth, 1986; Zwick, Budescu, and Wallsten, 1988; also see the survey of fuzzy set applications in psychology presented in Smithson and Oden, 1999). Similarly, in sociology and political science, fuzzy sets are used to enable 'diversity oriented' research and to strengthen the connection between theory and data analysis (Ragin 2000; Ragin and Pennings, 2005).

2.4 Application of Fuzzy Set Theory to the Economics of Inequality and Poverty

Amartya Sen appears to be the first to suggest that fuzzy set theory might be applied to the economics of inequality and poverty. Sen recognised (in his *On Economic Inequality*) that "the implicit notion of inequality that we carry in our mind is, in fact, much less precise ….. We may not indeed be able to decide whether one distribution *x* is more or less unequal than another, but we may be able to compare some other pairs perfectly well. …… There are reasons to believe that our idea of inequality as a ranking relation may indeed be inherently incomplete. If so, to find a measure of inequality that involves a complete ordering may produce artificial problems, because a measure can hardly be more precise than the concept it represents." (Sen 1973, p 5-6).

He has also made similar observations in the context of poverty. For example, in his *Poverty and Famines* he wrote that "while the concept of nutritional requirements is a rather loose one, there is no particular reason to suppose that the concept of poverty must itself be clear-cut and sharp. In fact, a certain amount of vagueness is implicit in both the concepts, and the really interesting question is the extent to which the areas of vagueness of the two notions, as commonly interpreted, tend to coincide. The issue, thus, is not whether nutritional standards are vague, but whether the vagueness is of the required kind." (Sen 1981, p 13).

Sen has indeed supported the use of fuzzy set theory and measures based on it, where precision of such measures is of importance. He writes: "A formal expression can be extremely precise without being at all a *precise representation* of the underlying concept to be captured. In fact, if that underlying concept is ambiguous, then the demands of precise representation call for *capturing* that ambiguity rather than replacing it by some different idea – precise in form but imprecise in representing what is to be represented. It is in this context that such mathematical structures as partial orderings, fuzzy sets, etc., have much to offer." (Sen 1989, p 317; italics as in the original).

While discussing inequality measurement, Kaushik Basu (1987) has argued that there are cases that fall between those where one can make a precise judgement and those where one could make no judgement at all – cases where one can only make an *imprecise* judgement. He writes: "Recent advances in the theory of fuzzy sets enable us to talk of human imprecisions in a meaningful way. And it is not difficult to maintain that a fuzzy binary relation captures our ambivalence in ranking states according to inequality better than a quasi-ordering" (Basu 1987, p. 276). It is on this basis that Basu has developed his axiomatic fuzzy set theoretic measure of inequality. One "interesting property" he has found is that "the conventional Gini-ranking is, in a sense, a best unfuzzy approximation of our fuzzy measure" (ibid,).

The use of fuzzy set theory has in no time naturally reached the area of poverty measurement with early contributions from Cerioli and Zani (1990) and Cheli and Lemmi (1995). As already explained above, fuzzy set theory as applied to poverty measurement, works in terms of a graded membership, suggesting a degree to which it is true that someone (or some household) is a member of the set of the poor. The membership function of the set of the poor is typically taken to lie on the [0,1] interval, with '0' suggesting definite non-membership, '1', definite membership and values between zero and one capturing the degree of membership. Formally, this 'membership function' maps an individual's (or household's) performance in terms of an indicator, or in terms of a set of indicators, on to a degree of membership of the set of the poor.

The first study of fuzzy poverty measurement (Cerioli and Zani 1990) has employed both a simple linear membership function, based on income only, and also some variations of multi-dimensional fuzzy poverty membership functions. In the simple case, they have taken a level of income below which a person (or household) is counted as definitely income poor, and another level above which she is judged to be definitely not income poor. Between these two levels, the degree of membership of the set of the poor linearly decreases as income increases. In the case of one of the alternative multi-dimensional measures, Cerioli and Zani have suggested an ordinal ranking of levels of disadvantage for each of the dimensions considered. In each dimension there is some level below which a person (or household) is taken as definitely poor, and another level above which she is classified as definitely not poor. Between these levels, the degree of membership of the set of the poor person (for each dimension) is based on her position in the ordinal ranking. Once this is done for all the dimensions, they have explored various ways of weighting the dimensions of poverty to derive an aggregate measure in order to judge whether or not a person (or household) is definitely poor considering all the dimensions of poverty. It must be noted that as long as each dimension has positive weight, a person (or household) must qualify as definitely poor with a score of 1 on all dimensions in order to be counted as definitely poor aggregately with an overall score of 1.

However, this methodology of arbitrarily using two critical levels to define the range of levels of income or other indicators of fuzziness has been criticized by Cheli and Lemmi (1995). They have instead suggested an alternative 'Totally Fuzzy and Relative' (TFR) approach. In this approach, the cut-offs used to mark the relevant range of levels of each indicator of fuzzy dimension is determined by the distribution itself. Only those persons (or households) who are most (least) deprived in terms of the distribution of the relevant indicator (such as income) are definitely poor (not poor) in terms of that indicator. Between these levels, the degree of membership of the set of the poor in the relevant indicator is based on the distribution of the relevant indicator. A large number of studies have employed the TFR approach; for example, Chiappero-Martinetti (1994, 1996, 2000), Lelli (2001); Qizilbash (2002) and Clark and Quizilbash (2003) use this method in order to analyse poverty or well-being in the context of Sen's capability approach.

Qizilbash (2002) has applied both the Cerioli-Zani and Cheli-Lemmi measures in the South African context, using data from the 1996 South African Census. The cutoffs used in this study for the fuzzy poverty measures are in line with the Cheli-Lemmi methodology. The worst-off (best-off) category in the sample is thus defined as definitely poor (definitely not poor) in each relevant dimension of the quality of life.

In another attempt to apply Sen's capability approach to the South African context, Stephan Klasen (1997; 2000) has used various indices as proxies for fourteen 'components' of his composite measure of deprivation (such as education, health, housing, nutrition, water, employment, safety, etc.). Each component is thought of as relating to some specific 'capability', with levels of achievement in terms of these components associated with a rank order number. Klasen has also included income as a component in his study. He has characterised the index which focuses only on the seven indices given above as a 'core deprivation index' (Klasen, 2000, p. 43). The choice of component indicators that Klasen has included in this index is motivated by the fact that they relate to capabilities listed in certain works by Amartya Sen (Klasen, 2000, p. 39).

Sara Lelli (2001) has been the first to try an empirical comparison, in the context of Sen's capability analysis, between the fuzzy sets approach (including the TFR) and factor analysis, a multivariate technique, preferred by some researchers; she has found that that the 'fuzzy aggregates' are insensitive to the choice of the form of the membership function. Lelli (2001, p. 25) has also found that both the methods (factor analysis and fuzzy sets) show that "income accounts only for a very limited part of the story and this should definitely be seen as a reason to follow multidimensional approaches like Sen's one."

Some intermediate contributions between Cerioli and Zani (1990) and the TFR approach are also present in the literature (for example, Dagum, Gambassi and Lemmi, 1992; Pannuzi and Quaranta, 1995; Blaszczk-Przybycinska. 1992). Dagum and Costa (2004) have developed an approach similar to TFR leading to the so called Dagum's decomposition (also see Mussard and Pi-Alperin, 2005).

The methodological implementation of the TFR approach has since then developed in two directions. The first one is typified by the contributions of Cheli (1995), Cheli and Betti (1999) and Betti *et al*. (2004), using the method to analyse the fuzzy concept of poverty in terms of transition probability matrices in the dynamic context of two consecutive panel data sets. The second, with the contributions of Betti and Verma (1999, 2002, 2004) and Verma and Betti (2002), has focused more on capturing the multi-dimensional aspects, developing the concepts of 'manifest' and 'latent' deprivation to reflect the intersection and union of different dimensions. A latest advance of this method is given by Betti *et al*. (2005) that combines the above two developments in the form of an *Integrated Fuzzy and Relative* (IFR) approach to the analysis of poverty and social exclusion.

Cornelissen et al., (2000) have developed a few fuzzy mathematical models to assess sustainable development based on context-dependent economic, ecological, and societal sustainability indicators. Although a decision-making process regarding sustainable development is subjective, they argue that fuzzy set theory links human expectations about development, expressed in linguistic propositions, to numerical data, expressed in measurements of sustainability indicators. The fuzzy models thus developed provide a novel approach to support decisions regarding sustainable development.

Von Furstenberg and Daniels (1991) and Baliamoune (2000) have applied fuzzy set theory to assess the degree of country compliance with the G-7 economic summit commitments. Baliamoune (2004) has been the first application of fuzzy-set theory to macroeconomic and social indicators of human well-being.

Buhong Zhenga and Charles Zheng (2015) propose to measure human development as a fuzzy concept. They stress that there exists a great deal of vagueness in quantifying a country's level of human development; one such source of vagueness is the weighting scheme embedded in the well-publicized UNDP's Human Development Index (HDI). They suggest to evaluate the resulting fuzziness in human development ranking with a truth value function. A truth value is simply a function of the probability that a randomly drawn bundle of weights will rank one country to have a higher HDI than another country. They derive simple and easily computable formulae for calculating the truth value. The method derived is equally applicable to fuzzy rankings with other composite indices.

2.5 Epilogue

Many concepts that we use in social sciences are not precise and crisp; they are all vague, fuzzy. Hence the emergence of fuzzy set theory to deal with them precisely and rigorously. Intensive applications of this theory are found profusely in many fields such as artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, robotics and of course in most of the 'soft' (social) sciences. The present chapter has provided a brief discussion on the development of these applications in general and also in particular in economics. We find that there is an increasing trend in the application of this theory in development economics. The objective of this Project also being the same, we now turn to the next chapter for a brief presentation of fuzzy set theory.

Chapter 3:
An Introduction to Fuzzy Set Theory

> Base step: A one day old human being is a child.
>
> Induction step: If an *n* day old human being is a child, then that human being is also a child when it is $n + 1$ days old.
>
> Conclusion: Therefore, a 36,500 day old human being is a child.

3.1 Prologue

This chapter presents a short discussion on the fuzzy set theory. We start with Aristotle's Logic and go onto the fuzzy logic in the next section. Following a description of the classical set theory, we introduce the fuzzy set theory, highlighting its distinguishing characteristic in terms of the membership function with some simple examples. Finally we discuss the most popular functions used for fuzzy set membership such as the linear, trapezoidal, and sigmoid function.

3.2 Logic and Fuzzy Logic

Aristotle was the first to develop Logic in Europe; in order to devise a concise theory of logic, and later mathematics, he posited the three 'Laws of Thought'. The third of these is the 'Law of the Excluded Middle', also known as the law of the excluded third (in Latin *principium tertii exclusi* or *tertium non datur* = no third (possibility) is given). The law states that out of two contradictory propositions (where one proposition is the negation of the other) one must be true, and the other false, or that every proposition must either be true or false: it will not be possible to be and not to be the same thing! History shows that Parminedes, a pre-Socratic Greek philosopher, had proposed the first version of this law around 400 BC, against which Heraclitus

had proposed that things could be simultaneously true and not-true – the first light on fuzziness. However, Plato is regarded as having laid the foundation of fuzzy logic; he indicated that there was a third region (beyond 'true' and 'false'), where these opposites "tumbled about". But this dimension of logic remained in the dark for many centuries, and it was only around 1920 that a systematic alternative to the bi-valued logic of Aristotle came to be proposed by Jan Łukasiewicz, a Polish logician and philosopher, who mathematically described a three-valued (or trinary) logic system in which there are three truth values indicating 'true', 'false' and some indeterminate third value, which might be translated as the term 'possible' and to which he assigned a numeric value between 'true' and 'false'. Later, he explored four-valued logics, five-valued logics, and then declared that in principle it is possible to derive an infinite-valued logic.
In 1921, Emil L. Post, an American mathematician, formulated additional truth degrees with more-than-two truth values ($n \geq 2$, where n are the truth values). Later on, Łukasiewicz and Alfred Tarski together formulated a logic on n truth values where $n \geq 2$ and in 1932 Hans Reichenbach introduced a logic of infinite truth values where n approaches infinity. It was on such infinite-valued logic that Zadeh based his fuzzy set theory and by extension fuzzy logic.

3.3 Set Theory – Classical

Classical set theory, also called *crisp* or naïve set theory, postulates that either an element belongs to the set or it does not. For example, for the set of integers, either an integer is even or it is not (that is, it is odd). Those objects that belong to a set are called its **members**. As objects we allow anything: numbers, people, other sets... If x is a member of A, then we write $x \in A$. (The symbol "$\in$" is a derivation of the Greek letter Epsilon, 'ε'.)

The membership or characteristic function of a crisp set may be written as

$$\mu_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$$

Some sets may be described in words, for example:

A is the set whose members are the first four positive integers.

B is the set whose members are the colors of the Indian flag.

By convention, a set can also be defined by explicitly listing its elements between braces (curly brackets), for example:

$C = \{4, 2, 1, 3\}$

$D = \{\text{saffron, white, green}\}$

Notice that two different descriptions may define the same set. For example, for the sets defined above, A and C are identical, since they have precisely the same members. The shorthand $A = C$ is used to express this equality. Similarly, for the sets defined above, $B = D$.

This method of listing elements is called Roster method or Enumeration or Description method.

Another method of using mathematical notation for describing a set by indicating the properties that its members must satisfy is called set-builder (or rule) notation or set comprehension.

The simplest sort of set-builder notation is $\{x : P(x)\}$, where P is a predicate in one variable. This indicates the set of everything satisfying the predicate P, that is, the set of every object x such that $P(x)$ is true. Some authors use the pipe symbol | rather than **:** to indicate the conditional. For example:

- $\{x : x \text{ is a real number and } x > 0\}$ is the set of all positive real numbers;

  $\{x \mid x \in \mathrm{R}\ ; x > 0\}$

- $\{k : \text{for some natural number } n, k = 2n\}$ is the set of all even natural numbers;

  $\{k \mid k = 2n\ ; n \in \mathrm{N}\}$

Also note that all sets under consideration in a certain case are subsets of some 'bigger' set, called universal set and denoted as 'U'. We also allow sets to be infinite and empty.

There are four common operations on sets: union, intersection, negation, and inclusion, denoted by the symbols $\cup$, $\cap$, $\sim$, and $\subset$, respectively. The first two are connectives, as they produce a new set from two or more sets under some given condition. Let us explain these operations using the two sets: $A = \{4, 2, 1, 3\}$ and $B = \{\text{saffron, white, green}\}$.

Union ($\cup$) combines two sets together; that is, it is the set of all things which are members of either A or B or both and corresponds to "or" in logic. With the two sets above, $A \cup B = \{4, 2, 1, 3, \text{saffron, white, green}\}$. Intersection ($\cap$) is the overlap between two sets; that is, it is the set of all things which are members of both A and B and corresponds to "and" in logic. The two sets above have no common elements and hence their intersection is empty. That is, $A \cap B = \emptyset$, the symbol for the null or empty set.

Negation (~), corresponding to 'not', gives the complement of a set, which contains all elements in the universal set that are not in the given set. Note that its definition requires that we define our universe (universal set, U) under consideration. of discourse, represented by the 'universal' set U. Suppose that for our Set A defined above, $U = \{\text{all counting numbers up to } 10\}$. Then $\sim A = \{5, 6, 7, 8, 9, 10\}$. Note that $A \cup \sim A = U$; that is, "everything that is A, and everything that is not A, is everything." Also note that $A \cap \sim A = \emptyset$; that is, "nothing is in both A and not A at the same time." This is the famous Law of the Excluded Middle, which the fuzzy set intersections do not generally obey.

Inclusion or containment ($\subset$) concerns whether a set includes/contains elements in another set. If every member of the set A is also a member of the set B, then A is said to be a subset of B, written $A \subset B$, and read as *A is contained in B*. Equivalently, we can write $B \supset A$, read as *B is a superset of A*, or *B includes A*, or *B contains A*. In the case of A and B given above, it is clear that neither set includes the other. However, given another set $C = \{1, 2, 3, 4, 5, 6\}$, we have $A \subset C$. Note that the asymmetry of inclusion is highly useful in examining relationships between empirical cases that are quite different from the correlations usually used in social sciences. Also note that inclusion and intersection have a special relationship: when $A \subset B$, then $A \cap B = A$ and when $A \subset B$ and $B \subset A$, then $A = B$.

3.4 Set Theory – Fuzzy

Now let us consider the set of vowels of the English letters, $V = \{a, e, i, o, u\}$. Since a letter is either a consonant or a vowel, logically the set of consonants is the complement of V, that is, $C = \sim V$. But, we know that in English, the letter *y* behaves strangely; it is sometimes a vowel and sometimes, a consonant. For example, in the word "my," *y* is a vowel, but in the word "your", it is not. Now the question is: Does *y* belong to set V or to set C? The answer is of course not definite as it belongs to both the sets, not just to any one. That is, the English letters cannot be classified into two mutually exclusive sets as implied by the dichotomy between vowels and consonants. This simply means that the letter *y* violates the Law of the Excluded Middle that is implied in the definition $C = \sim V$. Thus we have an issue of fuzziness here.

A fuzzy set is based on a classical set itself, but it is distinguished by its membership function that ranges from 0 to 1, that is, [0, 1], in contrast to that of the classical set constrained to either 1 or 0, that is, {0, 1}. Formally, given a set X of elements $x \in X$, any fuzzy subset A of X is defined as follows: $A = \{x, \mu_A(x)\}$, where $\mu_A(x)$: $X \rightarrow [0, 1]$, where [0, 1] is the interval of real numbers from 0 to 1, is called the *membership function* in the fuzzy subset A. The value $\mu_A(x)$ indicates the degree of membership of x in A, that is to say, the degree of truth of $A(x)$. In other words, the membership function is an index of "set-hood" that measures the degree to which an object x is a member of a particular set. Note that if $\mu_A(x) \in \{0, 1\}$, then A is an *ordinary* subset of X, which constrains the membership function to either 1 or 0. If $\mu_A(x) \in [0,1]$, then A is a *fuzzy* subset of X, which allows the membership function to range from 0 to 1. Also note that $\mu_A(x) = 0$ means that x does not belong to A, whereas $\mu_A(x) = 1$ means that x belongs to A completely. When $0 < \mu_A(x) < 1$, x partially belongs to A and its degree of membership in A increases in proportion to the proximity of $\mu_A(x)$ to 1. We can say that the membership function thus acts as a linear filter. In short, the main difference between classical set theory and fuzzy set theory is that the latter allows for partial set membership.

Let us consider an illustrative example in terms of the predicate 'young'. Here the universal set (P) includes all the people of different ages. We also define a fuzzy subset Y (for young) in terms of answers to the question 'to what degree is person x young?' Note that we have to assign a degree of membership in the fuzzy subset Y to each person in the universe. Obviously, the easiest way to do this is with a membership function based on the person's age. Thus we can define this membership function of a person x as follows:

$$\text{young}(x) = 1, \qquad \text{if age}(x) \le 20,$$
$$= \frac{[30-\text{age}(x)]}{30-20}, \quad \text{if } 20 < \text{age}(x) \le 30,$$
$$= 0, \qquad \text{if age}(x) > 30.$$

Given this definition, we can estimate some example values:

Table 3.1: Membership Value or Degree of Youth

Person	Age (years)	Degree of youth
Ahalya	10	1
Anasuya	15	1
Arundati	21	0.9
Damayanti	25	0.5
Renuka	26	0.4
Sakuntala	28	0.2
Sita	35	0
Urmila	60	0

Accordingly, we can say that the membership value of Damayanti is 0.5 or that the degree of truth of the statement 'Damayanti is young' is 0.5.

A graph of this membership function appears as follows:

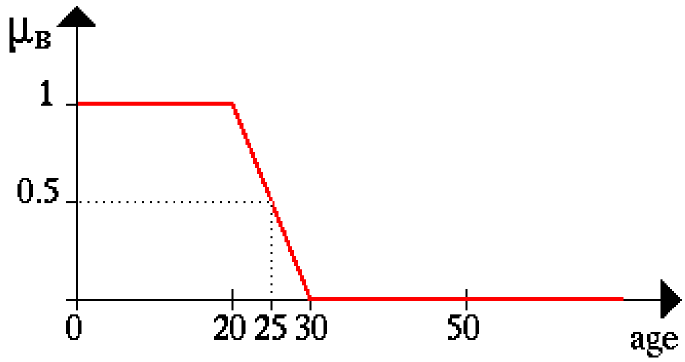


Fig. 3.1: Membership Function of Youth

For another example, let us consider the concept of poverty. In our definition given below, membership in the set of poor people is 1 if daily per capita expenditure of a rural person is below Rs. 27, the degree of membership decreases linearly for expenditures ranging from Rs. 27 up to Rs. 50, and equals 0 for any income that exceeds Rs. 50.

$$
\begin{aligned}
\text{Poor}(x) &= 1, && \text{if } x < 27,\\
&= \frac{50-x}{50-27}, && \text{if } 27 \le x \le 50,\\
&= 0, && \text{if } x > 50.
\end{aligned}
$$

Below we give some example values, based on this definition.

Table 3.2: Membership Value or Degree of Poverty		
Person	Expenditure (Rs)	Degree of poverty
Ahalya	10	1
Anasuya	26	1
Arundati	28	0.96
Damayanti	30	0.87
Renuka	35	0.65
Sakuntala	45	0.22
Sita	53	0
Urmila	60	0

It should be noted that probability and fuzziness are not the same; consider the following simple example that may make this clear. Suppose we have a statement: "element x belongs to a fuzzy subset with degree of membership of 0.5"; this is not the same as saying that x would belong to this set with a probability of 50%. If it turns out that x belongs to this set, then the membership value would just be 100%! Thus the probability measure, if taken as equivalent to fuzziness, takes away the notion of fuzziness altogether; i.e. it continues to operate within a binary setting.

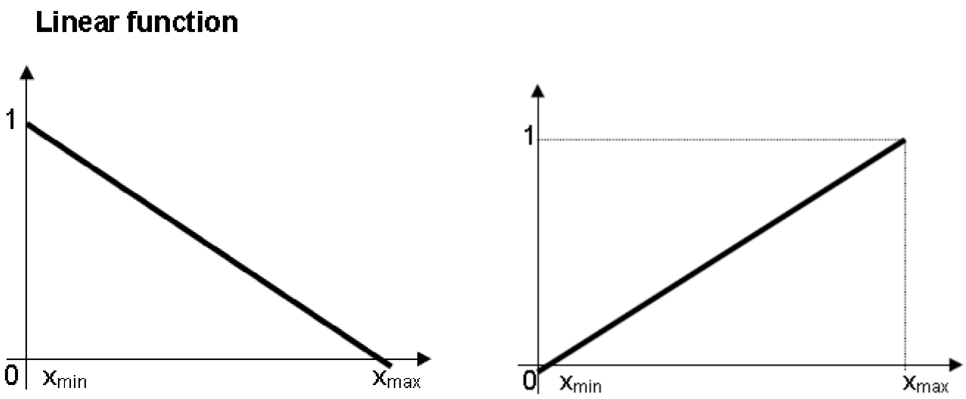
Note that the usual membership functions are not as simple as these cases. Hence next we consider the issue of measuring a membership function.

3.5 Measurement of Membership

We have already defined a membership function (in the fuzzy subset A of X) as $\mu_A(x)$: $X \rightarrow [0, 1]$, where [0, 1] is the interval of real numbers from 0 to 1, given a set X of elements $x \in X$. Hence, $\mu_A(x) = 0$ if the element $x \in X$ does not belong to A, $\mu_A(x) = 1$ if x completely belongs to A and $0 < \mu_A(x) < 1$ if x partially belongs to A.

Following Chiappero-Martinetti (2000), let us suppose that the subset *A* defines the position of a country according to the degree of achievement of a given development dimension (such as income, education, health, etc,). In this case, a membership value equal to one identifies a condition of full achievement of a given development dimension, while a value equal to zero shows the reverse condition (of poverty). The intermediate values between 0 and 1 then describe gradual positions within the spectrum, an ascent from complete poverty to complete development. Thus we find that in order to define a membership function, we have i) to define an appropriate arrangement of values on the basis of the different degrees of development ; ii) to identify the two extreme conditions, that is, $\mu_A(x) = 1$ (full membership) and $\mu_A(x) = 0$ (non-membership) ; and iii) to specify the membership values for all the other intermediate conditions.

Now, coming to the choice of a proper membership function, the main factors we have to consider are the application context and the kind of indicator that we want to describe. For example, in the case of variables with equi-distributed values along an ordinal scale (values such as 1, 2, 3, etc., or 10, 15, 20, etc.), we have to employ the linear functions given below (Chiappero-Martinetti 2000).



$$\mu(x) = \frac{X\max - X}{X\max - X\min} \qquad\qquad \mu(x) = \frac{X - X\min}{X\max - X\min}$$

Fig. 3.2: Linear Membership Functions

On the other hand, if it is possible to define, in clear terms, conditions of full membership (complete achievement) on one side and non-membership (total deprivation) on the other in the case of a given dimension of development such that it is possible to identify a given interval between these maximum and minimum threshold levels, then a trapezoidal function as shown below can be chosen (Chiappero-Martinetti 2000).

Trapezoidal function

$\mu(x) = 1$ if $x_{min} \leq x \leq z$

$\mu(x) = 0$ if $x_k \leq x \leq x_{max}$

$\mu(x) = \dfrac{x_k - x}{x_k - z}$ if $z \leq x \leq x_k$

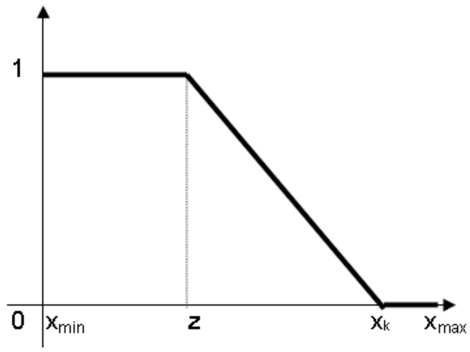


Fig. 3.3: Trapezoidal Membership Function

If we have quantitative and qualitative variables with values that are not equi-distributed, then a sigmoid function, shown below, seems appropriate to describe the relevant membership function (Chiappero-Martinetti 2000).

Sigmoid (or logistic) function

$x_k = \mu(x) = 0$ $x_h = \mu(x) = 0.5$ $x_w = \mu(x) = 1$

$\mu(x) = 1$ if $x_{min} \leq x \leq x_w$

$\mu(x) = 0$ if $x_k \leq x \leq x_{max}$

$\mu(x) = 1 - \dfrac{1}{2}\left[\dfrac{x_w - x}{x_w - x_h}\right]^2$ if $x_w \leq x \leq x_h$

$\mu(x) = \dfrac{1}{2}\left[\dfrac{x_k - x}{x_k - x_h}\right]^2$ if $x_h \leq x \leq x_k$

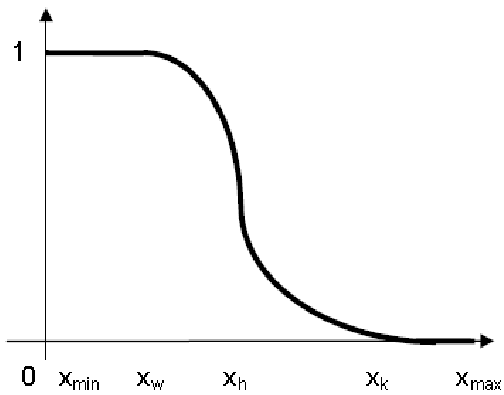


Fig. 3.4: Sigmoid (or Logistic) Membership Function

The present study employs the simple linear function to estimate the membership function in the context of development dimensions of selected States of India.

3.6 Epilogue

This chapter has discussed the main features of the fuzzy set theory, starting with the development of alternatives to the bi-valued logic of Aristotle and identifying Plato as having laid the foundation of fuzzy logic. With a description of the classical set theory, we have then paved the way for introducing the fuzzy set theory, highlighting its distinguishing characteristic in terms of the membership function with some simple examples. Finally we have discussed some of the most popular functions used for fuzzy set membership such as the linear, trapezoidal, and sigmoid function. Given this background, we now turn to the next chapter for analyzing development of the Indian States in the framework of fuzzy set theory.

Chapter 4

Data Analysis and Results

> "Mathematical objects generally can be defined precisely;
> empirical objects often cannot be so defined."
>
> – Smithson and Verkuilen (2006: 6)

4.1 Prologue

The present chapter constitutes the core of the study, presenting the data analysis and the results of this Project. We start with a brief description of the data resources and method of analysis, that is, the linear membership estimation in the framework of fuzzy set theory. After estimating the membership values for each state in the given development indicators, we also derive a global development index for each state considered in terms of an average of individual development indices.

Development is a multi-dimensional concept, but imprecise and incomplete in terms of the dimensions that reflect particular states of development. Even though theoretically it is possible to expand the list of identified possible dimensions, empirically data on these aspects might not be completely available for all the States. Hence we select only a few major States in India and a few main dimensions of development, where secondary data are readily available. The development dimensions that we consider are health, knowledge and standard of living. As already noted, these dimensions are latent factors, that is, unobservable; therefore, we use some indicators to proxy these development dimensions. The indicators of health dimension (along with the latest year for which data are available) are: (i) Life expectancy at birth (2006-10), (ii) Infant mortality rate (2012), (iii) Birth rate (2012), and (iv) Death rate (2012). As an indicator of knowledge, we take (v) Literacy Rate (2011), and that of standard of living, (vi) Per capita net state domestic product at constant (2004-05) prices (as on 31.10.2014); the last one represents average for three years from 2010-11, the three-year averaging for per capita net state domestic product has been carried out in order to iron out the possible fluctuations.

The selected states are: Andhra Pradesh, Assam, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal. As already noted, we have considered Bihar, Madhya Pradesh and Uttar Pradesh as undivided states, including the new states of Jharkhand (with Bihar), Chattisgarh (with Madhya Pradesh) and Uttarakhand (with Uttar Pradesh).

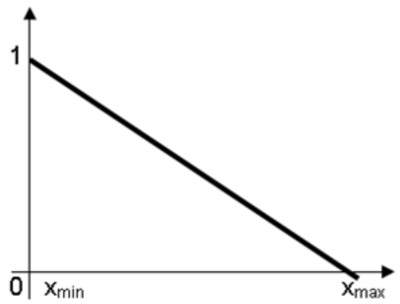
4.2 Method of Analysis

The data are analysed in the framework of fuzzy set theory. For want of acceptable benchmark values for all the indicators, we have used the maximum/minimum values in the data as cut-offs to identify the cases of definite membership and definite non-membership; the intermediate membership function values are derived using the simple linear model.

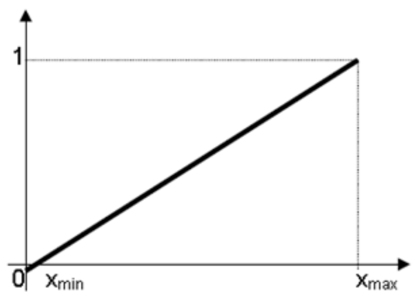
Note that we have two types of linear models (see below), decreasing and increasing; when we use (Xmax – X) as the numerator in the membership function, we have a decreasing linear function and when we use (X – Xmin) as the numerator in the membership function, we have an increasing linear function (in both the expressions, the denominator is the same: [Xmax – Xmin]). The deviation (Xmax – X) in the former case describes the degree of deprivation of a state in a given dimension X compared with the state with the maximum value in that dimension. Similarly, the deviation (X – Xmin) in the latter case describes the degree of achievement of a state in a given dimension X compared with the state with the minimum value in that dimension. Thus the former is a poverty (deprivation) measure and the latter is a development (achievement) measure. Note that the denominator serves to constrain (normalize) the measures in the range of zero to one, [0, 1]. Also note that development measure = one less deprivation measure, and deprivation measure = one less development measure. That is,

$$\frac{X-X_{\min}}{X_{\max}-X_{\min}} = 1 - \frac{X_{\max}-X}{X_{\max}-X_{\min}} \quad \text{or,} \quad \frac{X_{\max}-X}{X_{\max}-X_{\min}} = 1 - \frac{X-X_{\min}}{X_{\max}-X_{\min}}$$

We have to use one of these measures for a particular dimension, depending upon its nature, given our objective of analyzing development aspects.



$$\mu\,(x) = \frac{X\max - X}{X\max - X\min}$$



$$\mu\,(x) = \frac{X - X\min}{X\max - X\min}$$

For example, in the case of per capita net state domestic product we need an achievement-based membership function (with [X – Xmin] as the numerator); so are the cases of life expectancy and literacy. In these cases, one denotes full membership (in development) and zero, full non-membership, with linearly increasing intermediate values of graded membership of development (second figure above). But in the case of infant mortality rate, death rate and birth rate, where small X values denote development, we have to consider deprivation-based measure for development (with [Xmax – X] as the numerator). Here, one denotes full membership in development (or full non-membership in deprivation) and zero, full non-membership in development (or full membership in deprivation), with linearly decreasing intermediate values of graded membership of development (first figure above).

Once the membership values for each state in the given development indicators are estimated, a global development index for each state considered can be derived in terms of an average of individual development (indicator) indices:

$$D_s = \frac{1}{n}\sum_{i=1}^{n} \mu_s(x_i),$$

where D_s is the global development index for the state s and $\mu_s(x_i)$ is that state's membership value in the ith development indicator. The value of D_s may be taken to represent the proportion of people/households belonging in a fuzzy sense to the development subset in the state s.

4.3 The Results

The data that we use for the analysis on the six development indicators are reported in Table 4.1 below. To construct the membership functions of each state in these indicators, we have to identify the minimum and maximum values of each indicator which are presented in Table 4.2.

As already stated, per capita net state domestic product, life expectancy and literacy rate are positive development indicators (with higher values denoting higher levels of development), whereas infant mortality rate, death rate and birth rate are negative development indicators (with smaller values denoting higher levels of development). Table 4.2 shows that Maharashtra and Kerala stand distinguished as development leaders in that per capita net state domestic product is maximum and death rate is minimum for Maharashtra (and also for West Bengal); and literacy rate and life expectancy are maximum and infant mortality rate and birth rate are minimum for Kerala.

Table 4.1: State-wise Development Indicators						
	LEB	IMR	BR	DR	LR	PCNSDP
Andhra Pradesh	65.8	41	17.5	7.4	67	42390
Assam	61.9	55	22.5	7.9	72.2	22493
Bihar	65.8	43	27.7	6.6	64.1	19371
Gujarat	66.8	38	21.1	6.6	78	57493
Haryana	67	42	21.6	6.4	75.6	61216
Karnataka	67.2	32	18.5	7.1	75.4	41722
Kerala	74.2	12	14.9	6.9	94	52238
Madhya Pradesh	62.4	56	26.6	8.1	69.8	25199
Maharashtra	69.9	25	16.6	6.3	82.3	61758
Orissa	63	53	19.9	8.5	72.9	24511
Punjab	69.3	28	15.9	6.8	75.8	46319
Rajasthan	66.5	49	25.6	6.6	66.1	28392
Tamil Nadu	68.9	21	15.7	7.4	80.1	56320
Uttar Pradesh	62.7	53	27.4	7.7	73.25	34926
West Bengal	69	32	16.1	6.3	76.3	32456
India (average)	66.1	42	21.6	7	73	37702

Notes: LEB: Life expectancy at birth (years) 2006-10

IMR: Infant mortality rate (Total per 1000 live births) 2012

BR: Birth rate, DR: Death rate (per 1000) 2012

LR: Literacy Rate (percent) 2011

PCNSDP: Per capita net state domestic product at constant (2004-05) prices (in Rs.; average of three years from 2010-11; as on 31.10.2014)

Source: http://planningcommission.nic.in/data/datatable/

Table 4.2: Minimum and Maximum Values of Development Indicators				
	Minimum		Maximum	
	Value	State	Value	State
LEB	61.9	Assam	74.2	Kerala
IMR	12	Kerala	56	Madhya Pradesh
BR	14.9	Kerala	27.7	Bihar
DR	6.3	Maharashtra West Bengal	8.5	Orissa
LR	64.1	Bihar	94	Kerala
PCNSDP	19371	Bihar	61758	Maharashtra

Notes: LEB: Life expectancy at birth (years)

IMR: Infant mortality rate (Total per 1000 live births)

BR: Birth rate, DR: Death rate (per 1000)

LR: Literacy Rate (percent)

PCNSDP: Per capita net state domestic product at constant (2004-05) prices (in Rs.; average of three years from 2010-11; as on 31.10.2014)

Source: as above.

Based on Tables 4.1 and 4.2, we have estimated the state-wise development membership values, given in Table 4.3. As already explained in the section on Methods above, we use achievement-based membership function (with [X – Xmin] as the numerator) for the positive development indicators (per capita net state domestic product, life expectancy and literacy rate) and deprivation-based measure (with [Xmax – X] as the numerator) for the negative development indicators (infant mortality rate, death rate and birth rate). Below we illustrate the estimation method for life expectancy at birth (LEB; positive indicator) and infant mortality rate (IMR; negative indicator) for the state of Andhra Pradesh (AP).

For LEB, Maximum = 74.2 and Minimum = 61.9.

For IMR, Maximum = 56 and Minimum = 12.

For AP, LEB = 65.8 and IMR = 41.

Now the membership value of AP in LEB is given by

$$\frac{X - X_{\min}}{X_{\max} - X_{\min}} = \frac{(65.8 - 61.9)}{(74.2 - 61.9)} = \frac{3.9}{12.3} = 0.317,$$

and that in IMR is

$$\frac{X_{\max} - X}{X_{\max} - X_{\min}} = \frac{(56 - 41)}{(56 - 12)} = \frac{15}{44} = 0.341.$$

Similarly, for Kerala, the two measures are;

LEB: $\frac{X - X_{\min}}{X_{\max} - X_{\min}} = \frac{(74.2 - 61.9)}{(74.2 - 61.9)} = \frac{12.3}{12.3} = 1$, and

IMR: $\frac{X_{\max} - X}{X_{\max} - X_{\min}} = \frac{(56 - 12)}{(56 - 12)} = \frac{44}{44} = 1$.

Table 4.3: State-wise Development Membership Values						
	LEB	IMR	BR	DR	LR	PCNSDP
Andhra Pradesh	0.317	0.341	0.797	0.5	0.097	0.543
Assam	0	0.023	0.406	0.273	0.271	0.074
Bihar	0.317	0.295	0	0.864	0	0
Gujarat	0.398	0.409	0.516	0.864	0.465	0.899
Haryana	0.415	0.318	0.477	0.955	0.385	0.987
Karnataka	0.431	0.545	0.719	0.636	0.378	0.527
Kerala	1	1	1	0.727	1	0.775
Madhya Pradesh	0.041	0	0.086	0.182	0.191	0.137
Maharashtra	0.650	0.705	0.867	1	0.609	1
Orissa	0.089	0.068	0.609	0	0.294	0.121
Punjab	0.602	0.636	0.922	0.773	0.391	0.636
Rajasthan	0.374	0.159	0.164	0.864	0.067	0.213
Tamil Nadu	0.569	0.795	0.938	0.5	0.535	0.872
Uttar Pradesh	0.065	0.068	0.023	0.364	0.306	0.367
West Bengal	0.577	0.545	0.906	1	0.408	0.309
India (average)	0.341	0.318	0.477	0.682	0.298	0.432

Notes: LEB: Life expectancy at birth (years)

IMR: Infant mortality rate (Total per 1000 live births)

BR: Birth rate, DR: Death rate (per 1000)

LR: Literacy Rate (percent)

PCNSDP: Per capita net state domestic product at constant (2004-05) prices (in Rs.; average of three years from 2010-11; as on 31.10.2014)

Source: Computed from Table 4.1.

The estimated membership values of different states in the selected development indicators are given in Table 4.3 above. We find, as earlier (in Table 4.2), Kerala has full membership values (unity) in four out of six indicators, Maharashtra in two and West Bengal in one. Bihar lags far behind with full non-membership (zero) in three indicators, and Assam, Madhya Pradesh and Orissa have one zero each.

Table 4.4 reports the state-wise mean membership values, that is, the global development indices. Kerala stands first with an index of 0.917, followed by Maharashtra (0.805) and Tamil Nadu (0.701). It may be interpreted that in Kerala, 91.7 per cent of the people/households belong in a fuzzy sense to the development subset including the above six indicators, and in Maharashtra and Tamil Nadu, 80.5 per cent and 70.1 per cent of the people/households are 'developed' in a fuzzy sense. Note that the all-India average is only 0.425; also note that there are six states falling below this low average: Rajasthan, Bihar, Uttar Pradesh, Orissa, Assam, and Madhya Pradesh (see Table 4.5 below), the last one having the least mean development index; only 10.6 per cent of the people/households in this state belong to the development subset in a fuzzy sense.

Table 4.4: State-wise Global Development Indices (D_s)

State	D_s	State	D_s
Andhra Pradesh	0.432	Maharashtra	0.805
Assam	0.174	Orissa	0.197
Bihar	0.246	Punjab	0.660
Gujarat	0.592	Rajasthan	0.307
Haryana	0.589	Tamil Nadu	0.701
Karnataka	0.539	Uttar Pradesh	0.199
Kerala	0.917	West Bengal	0.624
Madhya Pradesh	0.106	India (average)	0.425

Table 4.5: States in Descending Order of Development		
Global Index Interval	States	Global Index
≥ 0.9	Kerala	0.917
0.8 to 0.9	Maharashtra	0.805
0.7 to 0.8	Tamil Nadu	0.701
0.6 to 0.7	Punjab	0.660
	West Bengal	0.624
0.5 to 0.6	Gujarat	0.592
	Haryana	0.589
	Karnataka	0.539
0.4 to 0.5	Andhra Pradesh	0.432
Below All-India Average	Rajasthan	0.307
	Bihar	0.246
	Uttar Pradesh	0.199
	Orissa	0.197
	Assam	0.174
	Madhya Pradesh	0.106

4.4 Epilogue

This core chapter has presented a discussion of the study analysis and its results. We have taken health, knowledge and standard of living as the development dimensions, the indicators of health dimension being life expectancy at birth, infant mortality rate, birth rate, and death rate; and the indicator of knowledge being literacy rate and that of standard of living, per capita net state domestic product at constant (2004-05) prices.The selected states are: Andhra Pradesh, Assam, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal. The results show that Kerala stands first in terms of the estimated development membership, followed by Maharashtra and Tamil Nadu; and that there are six states falling below a low all-India average: Rajasthan, Bihar, Uttar Pradesh, Orissa, Assam, and Madhya Pradesh.

Chapter 5
Conclusion

Who can turn a can into a cane?
Who can turn a pan into a pane?
It's not too hard to see,
It's Silent E.

Who can turn a cub into a cube?
Who can turn a tub into a tube?
It's elementary
For Silent E.

(Tom Lehrer's 1971 children's song "Silent E")

5.1 Prologue

It goes without saying that a large number of phenomena studied by social scientists, especially economists, involve vague predicates (such as rich, poor, developed) and nouns (such as poverty, development, welfare/wellbeing). However, they appear not to have been aware of such problem of vagueness, as we do not find any reference to it anywhere. What is more distressing is even after vagueness is captured in an exponentially growing literature on fuzzy set theory, its methodology is utilized only in a limited sphere of economics, that is, in poverty study. Most of the traditional methods used for poverty analysis are cursed with mainly two limitations: i) they are all uni-dimensional, in that they refer to only one proxy of poverty, namely, low income or low consumption expenditure; and ii) they need to divide the population into two exclusive categories of *the poor* and *the non-poor* by means of the so called *poverty line*. Now there is a strong recognition that poverty is a complex phenomenon and that it cannot be reduced to monetary dimension only. The advent of the fuzzy set theory has also helped reveal the vagueness of the concept such that it is now generally accepted that a multidimensional approach to poverty study is required involving a number of non-monetary indicators also of living conditions. Thus poverty, which is a manifestation of insufficient well-being, and hence development, which is a manifestation of sufficient well-being, are multidimensional phenomena and should therefore involve both monetary and non-monetary dimensions. It is true one can argue that with a higher income he may be able to improve some of his non-monetary dimensions of well-being. But there are things that his money cannot buy, just because they are simply not available; for example, a public good like flood control or malaria prevention program or a school or a hospital in an underdeveloped country. Hence it is essential that the monetary factor be supplemented with other non-monetary attributes, such as housing, literacy, life expectancy at birth, nutritional status, provision of public goods etc. This requirement for multi-dimensionality in turn also implies that information on a single attribute is not sufficient in the face of ambiguity in the concept of poverty/development and more information from diverse dimension is essential. Now, according to Amartya Sen, "If a concept has some basic ambiguity (as ideas of what constitutes 'inequality' tend to have), then a *precise* representation of that ambiguous concept must *preserve* that ambiguity, rather than try to remove it through some arbitrarily completed ordering. This issue is quite central to the need for *descriptive accuracy* in inequality assessment, which has to be distinguished from fully ranked, unambiguous assertions (irrespective of the ambiguities in the underlying concept)." (Sen 1997: 121). It is here that the fuzzy set theory of Zadeh (1965) plays a fundamental role in tackling problems arising from ambiguity. Thus, it goes without saying that the poverty status of a person or a nation is intrinsically fuzzy, as the concept of poverty itself is vague. This in turn justifies a fuzzy set approach to poverty measurement sufficiently.

5.2 Development, Capability Approach and Fuzzy Set Theory

It goes without saying that the term development is the opposite of poverty, and hence that concept also is vague and multi-dimensional. This term evades a unique definition and means different things to different economists. Todaro and Smith (2015: 18) emphasizes that development as a multidimensional process involves "major changes in social structures, popular attitudes, and national institutions, as well as the acceleration of economic growth, the reduction of inequality, and the eradication of poverty." According to them, "No one has identified the human goals of economic development as well as Amartya Sen, perhaps the leading thinker on the meaning of development." (*ibid.*).

Sen argues that it is not possible to properly measure poverty in terms of income or even by utility as usually understood; what matters fundamentally is not the things a person has—or the feelings these provide—but what a person *is,* or *can be*, and what she *does*, or *can do.* These beings and doings are called 'functionings', that is, what a person does (or can do and/or can be) with the commodities of given characteristics that she comes to possess or control. The valued beings and doings, or functionings that people have reason to value, can range from being healthy, being educated, being well-nourished, and well-clothed, to being mobile, having self-esteem, and "taking part in the life of the community." (Sen, 1985: 12). He then defines 'capability' as "the freedom that a person has in terms of the choice of functionings, given his personal features (conversion of characteristics into functionings) and his command over commodities." (Sen, 1985: 13). Thus alternative combinations of such functionings from which the individual can choose, in turn, define her capability. "Capability is thus a kind of freedom: the substantive freedom to achieve alternative functioning combinations (or, less formally put, the freedom to achieve various lifestyles)." (Sen 1999: 75)., or, "the range of options a person has in deciding what kind of life to lead." (Dreze and Sen 1995: 10-11).

An individual's freedom to promote the aspirations she has reason to value depends on her capability to achieve functionings that make up her wellbeing. In this sense, her freedom enhances with her capability set. Development is the process of enhancing freedom, expanding capability set, opportunities and choices "so that each person can lead a life of respect and value." (UNDP 2000: 2). In other words, "Development consists of the removal of various types of unfreedoms that leave people with little choice and little opportunity of exercising their reasoned agency. The removal of substantial unfreedoms, …, is *constitutive* of development" (Sen 1999: xii). These freedoms are both the primary ends and principal means of development (Sen 1999: 10).

Sen's capability approach helps explain why development economists have placed so much emphasis on health and education, and more recently on social inclusion and empowerment, and why they have referred to countries with high levels of income but with poor conditions of health and education as cases of 'growth without development' (See, for example, Easterly 2003).

5.3 Development in India – The Study Problem

Even though India has recently become one of the fastest growing economies of the world and one among the most important G-20 economies, in terms of many development indicators, India has not fared well. Ours is a country of wide diversity in regional, social, economic, political, and cultural dimensions. As Jawaharlal Nehru, India's first prime Minister, stated in his *Discovery of India*: "India is a geographical and economic entity, a cultural unity amidst diversity, a bundle of contradictions held together by strong but invisible threads." (Nehru [1946] 1999: 562). Different States with different policy mixes have witnessed very different outcomes over the years. Some States have focused only on growth and some States have won laurels in achieving the objectives of both growth and development simultaneously. Analysis of this diversity and disparity across the States in their performance would help us identify useful policies of development. Hence the present study makes a novel attempt to analyse the issues of important aspects of development on a comparative plane across the Indian States.

However, as we have already seen, many concepts/predicates, such as poverty (or poor) and its opposite, development (or developed), used in economics are both vague/fuzzy and multi-dimensional and their analysis requires careful consideration of a graded membership. This study therefore employs the framework of fuzzy set theory in identifying and analysing the positions of different states in the development ladder, that is, their graded memberships in each development dimension and in aggregation. In particular, the objectives of the study are (i) to review the methodologies of classical set theory and fuzzy set theory; (ii) to identify the possible and empirically available dimensions of development of the Indian States; and (iii) to analyse these important dimensions of development on a comparative plane across the Indian states in the framework of fuzzy set theory.

5.4 Development Dimensions and Data

As we have already seen, development is a multi-dimensional concept, but imprecise and incomplete in terms of the dimensions that reflect particular states of development. Even though theoretically it is possible to expand the list of identified possible dimensions, empirically data on these aspects might not be completely available for all the states. Hence we select only a few major states and a few main dimensions of development, where secondary data are readily available. The development dimensions that we consider are health, knowledge and standard of living. Note that these dimensions are latent factors, that is, unobservable; hence we have to use some indicators to proxy these development dimensions. The indicators of health dimension are (along with the latest year for which data are available): (i) Life expectancy at birth (2006-10), (ii) Infant mortality rate (2012), (iii) Birth rate (2012), and (iv) Death rate (2012). As an indicator of knowledge we take literacy rate (2011), and that of standard of living, per capita net state domestic product at constant prices(2004-05) prices (as on 31.10.2014); the last one represents average for three years from 2010-11. We have used a three-year average for per capita net state domestic product in order to iron out the possible fluctuations.

The selected states are: Andhra Pradesh, Assam, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal. The data have been sourced from the Planning Commission of India (http://planningcommission.nic.in/data/datatable/).

5.5 Method of Analysis and Results

The data are analysed in the framework of fuzzy set theory. A fuzzy set has as its base a classical set itself, but what distinguishes the two is the membership function, which *ranges* from 0 to 1 in the case of the fuzzy set, whereas the membership function of the classical set is constrained to either 1 (full membership) or 0 (non-membership). In the former (fuzzy set) case, degree of membership increases in proportion to its proximity to 1. Thus, the main difference between classical set theory and fuzzy set theory is that the latter allows for graded set membership. The present study is an attempt to estimate the partial membership function of different States in India in respect of a number of development indicators. For want of acceptable benchmark values for all the indicators, we have used the maximum/minimum values in the data as cut-offs to identify the cases of definite membership and definite non-membership; the intermediate membership function values are derived using the simple linear model.

The results of the analysis show that Maharashtra and Kerala stand distinguished as development leaders; per capita net state domestic product is maximum and death rate is minimum for Maharashtra (and also for West Bengal); and literacy rate and life expectancy are maximum and infant mortality rate and birth rate are minimum for Kerala. Thus, in terms of the estimated state-wise development membership values, Kerala has full membership values (unity) in four out of six indicators, Maharashtra in two and West Bengal in one. Bihar lags far behind with full non-membership (zero) in three indicators, and Assam, Madhya Pradesh and Orissa have one zero each.

In respect of the state-wise mean membership values, that is, the global development indices, Kerala stands first with an index of 0.917, followed by Maharashtra (0.805) and Tamil Nadu (0.701). This mean membership figure may be interpreted in terms of, say, 91.7 per cent of the people/households in Kerala as belonging in a fuzzy sense to the development subset including the above six indicators. However, the all-India average comes out to be only 0.425; and also there are six states falling below this low average: Rajasthan, Bihar, Uttar Pradesh, Orissa, Assam, and Madhya Pradesh, the last one having the least mean development index; only 10.6 per cent of the people/households in this state belong to the development subset in a fuzzy sense.

5.6 Comparison with UNDP's HDI

The distinguishing feature of the present study, compared with the UNDP's HDI, is that it accepts the concept of development as fuzzy; moreover, it takes up more number of development indicators.

The UNDP, in fact, uses a normalization method for the dimension indices, similar to one of the linear membership functions of fuzzy set theory, even though the UNDP does not subscribe to viewing the concept of human development as fuzzy.

The present study considers the same development dimensions as the UNDP: health, knowledge and standard of living. The latter has however taken only life expectancy as the indicators of health dimension, whereas the present study has considered the following indicators: Life expectancy at birth, Infant mortality rate, Birth rate, and Death rate. The present study assumes that literacy rate is sufficient as an indicator of knowledge, rather than a weighted average of both literacy rate and enrolment ratio, as in the UNDP. As an indicator of standard of living, the present study takes per capita net state domestic product at constant (2004-05) prices, averaged for three years from 2010-11, in order to iron out the possible fluctuations, whereas the UNDP considers only one annual figure.

5.7 Possible Directions for Further Research

As already stated, fuzzy set theory involves immense scope for further studies of development dimensions; for one instance, instead of using the simple linear membership function as in the present study, it is possible to employ or even to develop more appropriate functions. In micro level poverty studies, with primary survey on household/individual level multidimensional deprivations, or on operationalizable indicators of Amartya Sen's Capability Theory, the role of fuzzy set approach is significant; moreover, this approach is very much useful in macro level studies also with both primary and secondary data.

5.8 Epilogue

The present study has made a novel attempt to analyse the issues of important aspects of development on a comparative plane across the Indian States in terms of an application of fuzzy set theory. The main limitation of the study comes from the non-availability of data on all the possible dimensions of development for all the States in India. Depending upon the available data, we have considered only six dimensions of development and 15 major States in India.

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